Section 1

Directions: Solve the following problems. Do not spend too much time on any one problem.

Note:
1. Unless otherwise specified, the domain of any function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.
2. $i$ will be used to denote $\sqrt{-1}$.
3. Figures that accompany the following problems are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated.

1. Which of the following numbers is a prime number?
   
   (A) $\sqrt{2}$
   (B) 119
   (C) $N^2$ ($N$ is any natural number)
   (D) $\log_327$

2. Which of the following numbers is a rational number?
   
   (A) $\frac{2}{\sqrt{3}}$
   (B) $\log_32$
   (C) $\frac{2}{e}$
   (D) $N$ ($N$ is any natural number)

3. Let $N$ be any natural number. Which of the following numbers is an odd number?
   
   (A) The sum of an even number and an odd number
   (B) $N^2$
   (C) $N^2 \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{2}{3} f(x) = \frac{x}{x^2 - 3x - 4} + 2N$
   (D) The product of an even number and an odd number
4. Let the \( f(x) = \frac{x}{x^2 - 3x - 4} \). Which of the following is the domain of \( f(x) \)?

(A) \( x \neq 4 \)
(B) All real numbers
(C) \( x \neq 4 \) and \( x \neq -1 \)
(D) \( x > 4 \)

5. Let the function \( f(x) = |x| + 1 \). Which of the following is the range of \( f(x) \)?

(A) \( x \geq 0 \)
(B) \( x \geq 1 \)
(C) All real numbers
(D) All positive real numbers

6. \( A = \{1, 2, 3, 4\} \)
\( B = \{4, 5, 6, 7\} \)

Which of the following sets is the union of \( A \) and \( B \)?

(A) \( \{4\} \)
(B) \( \{1, 2, 3, 4\} \)
(C) \( \{1, 2, 3, 4, 4, 5, 6, 7\} \)
(D) \( \{1, 2, 3, 4, 5, 6, 7\} \)

7. \( A = \{1, 2, 3, 4\} \)
\( B = \{4, 5, 6, 7\} \)

Which of the following sets is the intersection of \( A \) and \( B \)?

(A) \( \{4\} \)
(B) \( \{1, 2, 3, 4\} \)
(C) \( \{1, 2, 3, 4, 4, 5, 6, 7\} \)
(D) \( \{1, 2, 3, 4, 5, 6, 7\} \)
8. \[ A = \{1, 2, 3, 4\} \]
\[ B = \{4, 5, 6, 7\} \]

Which of the following sets is a subset of A?

(A) Union of A and B
(B) Intersection of A and B
(C) Set B
(D) \{0\}

9. Let function \( g(x) = x + 1 \), function \( f(x) = \frac{(x+1)}{x} \). Which of the following is the domain of the compound function \( \frac{f(x)}{g(x)} \)?

(A) \( x \neq -1 \)
(B) \( x \neq 0 \)
(C) \( x \neq 1 \) and \( x \neq 0 \)
(D) All real numbers

10. Let function \( g(x) = 2x - 1 \), and function \( f(x) = x^2 + 1 \). Which of the following numbers is equal to \( f(g(-1)) \)?

(A) 2
(B) 5
(C) -8
(D) 10

11. Let set \( A = \{1, 2, 3\} \), set \( B = \{1, 3, 5\} \), and set \( C = \{2, 4, 6\} \); and let \( \cap \) represent intersection, and \( \cup \) represent union. Which of the following sets is equal to \( (A \cap B) \cup C \)?

(A) \( \{1, 3, 4, 6\} \)
(B) \( \{1, 2, 3, 4, 6\} \)
(C) \( \{1, 2, 3, 4, 5, 6\} \)
(D) \( \{2, 4\} \)
12. Let \( \cap \) represent intersection, and \( \cup \) represent union. Which of the following expressions represents the next Venn diagram?

(A) \((A \cap B) \cup C\)
(B) \(A \cap B \cap C\)
(C) \((A \cap B) \cup (C \cap B)\)
(D) \((A \cap C) \cup (A \cap B)\)

13. Simplify the expression \( \frac{1 - \frac{1}{x^3}}{1 - \frac{1}{x^2}} \). Then, which of the following expressions is the simplified result?

(A) \(\frac{1}{x}\)
(B) \(1 + \frac{1}{x} + \frac{1}{x^2}\)
(C) \(\frac{x^2 + x + 1}{x^2 + x}\)
(D) \(1 - \frac{1}{x}\)

14. Let set \(A = \{1, 2, 3\}\) and set \(B = \{a, b\}\). Then, which of the following correspondences is NOT a function?

(A) \[1 \rightarrow a \quad 2 \rightarrow b \quad 3 \rightarrow a\]
(B) \[1 \rightarrow a \quad 2 \rightarrow b \quad 3 \rightarrow a\]
(C) \[2 \rightarrow b \quad 3 \rightarrow b\]
(D) \[1 \rightarrow a \quad 2 \rightarrow b \quad 3 \rightarrow a\]
15. Let \( y = x^2 + 1 \), then which of the following graphs represents \( y \)?

(A) \[ y = x^2 + 1 \]

(B) \[ y = x^2 + 1 \]

(C) \[ y = x^2 + 1 \]

(D) \[ y = x^2 + 1 \]

16. Let the following pairs of linear equations represent two lines. Which of the following pairs of lines is parallel?

(A) \[ x + 2y = 1 \] and \[ 2x + 3y = 5 \]

(B) \[ 6x + 2y = 5 \] and \[ x + \frac{1}{3}y = 2 \]

(C) \[ x - 2y = 3 \] and \[ 2x = y + 1 \]

(D) \[ x + 3y = 1 \] and \[ -3x + y = 2 \]

17. Let \( N \) be any natural number. Let set \( A \) be any number that can be written as \( N^2 \) and let set \( B \) be any number that can be written as \( \frac{1}{N} \). An ordered pair \((a, b)\) represents the Cartesian Product of \( A \times B \). Then, which one of the following ordered pairs is a member of the Cartesian Product of \( A \times B \)?

(A) \((4, 2)\)

(B) \((3, \frac{1}{4})\)

(C) \((4, 0.2)\)

(D) \((5, 1)\)
18. Let $a$ represent “$3x + 4 = 10$” and $s$ represent “$9x - x = 16$.” If $a \land s$ is true, which of the following could be the value of $x$?

(A) $-1$
(B) $0$
(C) $2$
(D) $5$

19. Let function $f(x) = |2x - 5|$. Which of the following numbers is equal to $f(-2)$?

(A) $9$
(B) $1$
(C) $-9$
(D) $-1$

20. Let $y = x + 1$ and $y = 0.5x$ be two lines, which are indicated in the following graph. Which of the following inequalities represents the shaded area?

(A) $y < x + 1$ and $y < 0.5x$
(B) $y > x + 1$ and $y > 0.5x$
(C) $y < x + 1$ and $y > 0.5x$
(D) $y > x + 1$ and $y < 0.5x$
21. Let \( n > m > 0 \). Then among \( \frac{n}{m}, \frac{m}{n}, \frac{n^2}{m^2}, \) and \( \frac{m^2}{n^2} \), which of the following expressions represents the largest number?

(A) \( \frac{n}{m} \)

(B) \( \frac{m}{n} \)

(C) \( \frac{n^2}{m^2} \)

(D) \( \frac{m^2}{n^2} \)

22. If \( |x - 2| < 5 \), which of the following statements is TRUE?

(A) \( x > 7 \)

(B) \( x < -3 \)

(C) \( x < -3 \) or \( x > 7 \)

(D) \( -3 < x < 7 \)

23. In base two, which of the following expressions is equal to \( 1110 - 111 \)?

(A) 111

(B) 101

(C) 1000

(D) 110

24. Let \( A = 5 \), and \( B = 128 \), in base two. Which of the following numbers in base two is equal to \( AB \)?

(A) 1,110,000,000

(B) 1,010,000,000

(C) 101,000,000

(D) 111,000,000
25. Let \( A \) and \( B \) be prime numbers, with \( A \neq 2, B \neq 2 \). Which of the following expressions is always a composite number?

(A) \( (A \times B)^{0.5} \)

(B) \( \frac{A}{B} \)

(C) \( A + B \)

(D) \( A - B \)

26. Let \( N \) be a natural number other than 1. Which of the following expressions is always divisible by a natural number?

(A) \( N + 1 \)

(B) \( N^2 + 1 \)

(C) \( N^3 + 1 \)

(D) \( N^4 + 1 \)

27. Which of the following reflects the interval \(-7 \leq y \leq 3\)?

(A) \( (-7, 3) \)

(B) \( [-7, 3) \)

(C) \( [-7, 3] \)

(D) \( (-7, 3] \)

28. Which of the following normal distributions has the smallest standard deviation?
29. Which of the following represents the mode of the data shown in the graph?

![Graph showing number of people (in thousands) and parades]

(A) 100  
(B) 200  
(C) 400  
(D) 500

30. Which of the following expressions is the standard form of \(\dfrac{(1+i)}{1-i}\)?

(A) \(\dfrac{(1+i)^2}{2}\)  
(B) \(i\)  
(C) \(-i\)  
(D) 1
Section 2

**Directions:** Solve the following problems. Do not spend too much time on any one problem.

**Note:**
4. Unless otherwise specified, the domain of any function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.
5. $i$ will be used to denote $\sqrt{-1}$.
6. Figures that accompany the following problems are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated.

31. If $m\angle ACB = 67^\circ$, then what is the measure of $a$?

(A) $19^\circ$
(B) $27^\circ$
(C) $33^\circ$
(D) $34^\circ$

32. Let $y = x(x^2 - 1)$. Then which of the following statements is always TRUE?

(A) If $x < -1$, then $y < 0$.
(B) If $x < 0$, then $y > 0$.
(C) If $x > 0$, then $y > 0$.
(D) If $x > 1$, then $y < 0$. 
33. Let \( 2^x = 16^{x-1} \). Which of the following is the solution of the equation?

(A) \( 1 + i \) and \( 1 - i \)
(B) 2
(C) \( \frac{1}{2} + 3i \) and \( \frac{1}{2} - 3i \)
(D) 1 and –1

34. Let \( x^2 = 25 \) and \( y^3 = 64 \). Then which of the following inequalities is TRUE?

(A) \( x > y \)
(B) \( x < y \)
(C) \( x = y \)
(D) None of the above

35. Let \( \log_{25} X = \frac{1}{2} \). Then which of the following is equal to \( X \)?

(A) 50
(B) 10
(C) 5
(D) 1

36. Let \( y = a^x \), \( y = b^x \), and \( y = c^x \) be three exponential functions; their graphs are as follows. Then which of the following expressions is TRUE?

(A) \( a < c < b \)
(B) \( a < b < c \)
(C) \( b < c < a \)
(D) \( c < b < a \)
37. Let \( \log_a X = 2 \) and \( \log_a b = 5 \). Then which of the following is equal to \( \log_b X^{-\frac{1}{2}} \)?

(A) \( -\frac{1}{10} \)

(B) \( -\frac{1}{5} \)

(C) \(-10\)

(D) \(-53\)

38. The following is a partial Truth Table:

\[
\begin{array}{ccc}
 p & q & r \\
 T & T & F \\
 T & F & T \\
\end{array}
\]

According to this partial Truth Table, which of the following expressions correctly describes \( r \)?

(A) \( p \) and \( q \)

(B) \( p \) and \( \neg q \)

(C) \( \neg p \) and \( q \)

(D) \( \neg p \) and \( \neg q \)

39. Triangle \( ABC \) is an isosceles right triangle with side lengths of 4 inches. What is the length, in inches, of the hypotenuse?

(A) 2

(B) \( 2\sqrt{3} \)

(C) \( 4\sqrt{2} \)

(D) 15

40. Which of the following statements is the inverse statement of “If \( A \) is true, then \( B \) is TRUE”?

(A) If \( B \) is true, then \( A \) is true.

(B) If \( A \) is false, then \( B \) is false.

(C) If \( B \) is false, then \( A \) is false.

(D) If \( B \) is true, then \( A \) is false.
41. The statement “The weather is good and both Charles and John go to the movies” is the sufficient and necessary condition of the statement “Linda will go with either Charles or John.” Then, if Linda did not go to the movies, which of the following statements must be TRUE?

(A) Charles did not go.
(B) Charles and John did not go.
(C) The weather is bad, and both Charles and John did not go to the movies.
(D) The weather is bad, or Charles did not go to the movies, or John did not go to the movies.

42. Let \(-A\) represent not \(A\), \(A \land B\) represent \(A\) and \(B\), and \(A \lor B\) represent \(A\) or \(B\). Let \(A\) and \(B\) be two logic variables: \(A = \text{true}, B = \text{false}\). Which of the following statements is always TRUE?

(A) \((-A) \land (B)\)
(B) \(- (A \land B)\)
(C) \(((A) \land (B) (A \land (-B)))\)
(D) \(((A) \lor (B) (A \lor (-B)))\)

43. Let \(-A\) represent not \(A\), \(A \land B\) represent \(A\) and \(B\), \(A \lor B\) represent \(A\) or \(B\). The following is a Truth Table.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

According to this Truth Table, which of the following expressions correctly describes the Truth Table?

(A) \((-A) \land B\)
(B) \(- (A \land B)\)
(C) \(((A) \land (B) (A \land (-B)))\)
(D) \(((A) \lor (B) (A \lor (-B)))\)

44. Let \(A\) be a sufficient condition for \(B\); and \(B\) a necessary condition for \(C\). Which of the following statements is TRUE?

(A) \(B\) is a necessary condition of \(A\).
(B) \(A\) is a sufficient condition of \(C\).
(C) \(A\) is a necessary condition of \(C\).
(D) \(B\) is a sufficient condition of \(C\).
45. Suppose a specific 4-digit number is composed only of 1, 2, 3, and 4. Each of 1, 2, 3, and 4 can be used only once in a number. Then how many different 4-digit numbers can be composed?

(A) 48
(B) 24
(C) 12
(D) 6

46. Suppose we use the letters A, B, C, D, and E to compose words which are 3 letters long and can only have three different letters. Then how many different words can be composed?

(A) 120
(B) 60
(C) 24
(D) 12

47. Suppose we select 3 people from 6 candidates. Then how many different ways can we select the 3 people?

(A) 2
(B) 10
(C) 20
(D) 12

48. In the figure shown, \( q \perp p \). What is the value of \( x \)?

(A) 18°
(B) 32°
(C) 47°
(D) 50°
49. Which of the following is a factor of $7x^2 - 12x - 4$?

(A) $x - 2$

(B) $x - 3$

(C) $x + 4$

(D) $x + 7$

50. Let a sample set be $\{-3, -1, 1, 2, 4, 6, 19\}$. Then, what is the difference between mean and median of this sample set?

(A) 2

(B) 1.5

(C) 3

(D) 1

51. Suppose we have 6 balls—labeled as $A$, $B$, $C$, $D$, $E$, and $F$—and each ball has the same probability to be picked. If we randomly pick 2 balls, which of the following is the probability of ball $A$ being picked?

(A) $\frac{1}{3}$

(B) $\frac{1}{4}$

(C) $\frac{1}{5}$

(D) $\frac{1}{6}$

52. Suppose we have 6 balls—labeled as $A$, $B$, $C$, $D$, $E$, and $F$—and we first pick one ball, do not put it back, then pick a second one. Which of the following is the probability of picking the $A$ ball on the second try?

(A) $\frac{1}{3}$

(B) $\frac{1}{4}$

(C) $\frac{1}{5}$

(D) $\frac{1}{6}$
53. A fair die is tossed. Which of the following is the probability that the die shows 4 points?

(A) \( \frac{2}{3} \)

(B) \( \frac{1}{3} \)

(C) \( \frac{1}{6} \)

(D) \( \frac{1}{2} \)

54. A fair die is tossed. Which of the following is the probability that the die shows no more than 4 points?

(A) \( \frac{5}{6} \)

(B) \( \frac{1}{3} \)

(C) \( \frac{1}{2} \)

(D) \( \frac{2}{3} \)

55. Which of the following expressions is equal to the number of diagonals of a polygon with \( N \) sides if \( N > 2 \)?

(A) \( 0.5N^2 - 0.5N \)

(B) \( 0.5N(N-1) - N \)

(C) \( N^2 - N \)

(D) \( 2N \)

56. Suppose one person has a 90% chance to hit a target with each shot, and all shots are independent of one another. What is the probability of hitting the target at least twice in three shots?

(A) 0.972

(B) 0.243

(C) 0.729

(D) 0.001
57. Let $A$, $B$ be two fair dice. First $A$ is tossed, then $B$ is tossed.

What is the probability of die $A$ showing 1, and die $B$ showing 1?

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{36}$

(D) $\frac{1}{34}$

58. Let $A$, $B$ be two fair dice. First $A$ is tossed, then $B$ is tossed.

What is the probability that the sum of the points showed by die $A$ and die $B$ is equal to 5?

(A) $\frac{1}{9}$

(B) $\frac{5}{36}$

(C) $\frac{1}{6}$

(D) $\frac{1}{18}$

59. Let $S$ represent the sum of the points showed by die $A$ and die $B$ and let $P(S)$ represent the probability of the sum. Which of the following probabilities is the largest?

Let $A$, $B$ be two fair dice. First $A$ is tossed, then $B$ is tossed.

(A) $P(3)$

(B) $P(7)$

(C) $P(9)$

(D) $P(11)$
60. Suppose we have an instrument with 10 components. If any one of the 10 components fails, then the instrument will not work. Let each component have 0.95 probability of being in working condition. Then what is the most accurate probability that the instrument works?

The following values may be used in calculation, in case you have no calculator:

\[
\begin{align*}
0.95^2 &= 0.9 \\
0.95^3 &= 0.86 \\
0.95^4 &= 0.81 \\
0.95^5 &= 0.77
\end{align*}
\]

(A) 0.8  
(B) 0.7  
(C) 0.6  
(D) 0.5
1. The correct answer is D. A prime number is a natural number greater than 1 that has no factors other than itself and 1. Since \( \sqrt{2} \) is an irrational number, then choice A is wrong; \( 119 = 7 \times 17 \), so choice B is wrong; when \( N = 1 \), \( N^2 = 1 \) is not a prime or a composite number; in all other cases \( N^2 \) is a composite number. \( \log_3 27 = \log_3 3^3 = 3 \), which is a prime number.

2. The correct answer is D. A rational number can be expressed as an integer divided by a non-zero integer. All natural numbers are rational numbers, so choice D is correct. All the other answers are not.

3. The correct answer is A. Let \( 2N \) be an even number, \( 2N + 1 \) an odd number. Then \( 2N + 2N + 1 = 4N + 1 \), which is still an odd number. Then, choice A is correct. Both \( N^2 \) and \( N^2 + 2N \) may be an odd number or an even number. The product of an even number and an odd number is always an even number.

4. The correct answer is C. The denominator of \( f(x) \) cannot be 0, thus
\[
x^2 - 3x - 4 = (x - 4)(x + 1) \\
\neq 0
\]
\( x \neq 4 \) and \( x \neq -1 \).

5. The correct answer is B. Since \( |x| \geq 0 \) is always true, \( |x| + 1 \geq 0 + 1 \), and \( f(x) \geq 1 \).

6. The correct answer is D. The union of A and B should include all elements in A, B; then, choices C and D satisfy this requirement, but element 4 can be counted only once, so then choice D is correct.

7. The correct answer is A. The intersection of A and B should include all common elements in set A and set B, so element 4 is the only common element; hence, choice A is correct.

8. The correct answer is B. The intersection of A and B should include all common elements in set A and set B; thus, the elements of the intersection of A and B must be a subset of A.

9. The correct answer is C. The domain of \( f(x) \) is \( x \neq 0 \), and the compound function \( \frac{f(x)}{g(x)} \) requires \( g(x) \neq 0 \); thus, \( x + 1 \neq 0 \), thus \( x \neq -1 \). The domain of the compound function is \( x \neq -1 \) and \( x \neq 0 \).

10. The correct answer is D. Since \( g(-1) = 2(-1) - 1 = -3, f(g(-1)) = f(-3) = (-3)^2 + 1 = 9 + 1 = 10 \); thus, choice D is correct.

11. The correct answer is B. Since \( A \cap B = \{1, 3\} \), and \( \{1, 3\} \) union with set C is \( \{1, 3, 2, 4, 6\} \), the correct answer is choice B.

12. The correct answer is C. The shaded area should consist of \( A \cap B \) and \( C \cap B \). Then choice C includes both sets. All the other answers fail to include both sets completely.
13. The correct answer is C.

\[
1 - \frac{1}{x^3} = \frac{1 - \frac{1}{x^2}}{(x^3 - x)}
\]

\[
= \frac{(x - 1)(x^2 + x + 1)}{(x(x - 1)(x + 1))}
\]

\[
= \frac{x^2 + x + 1}{(x^2 + x)}
\]

Therefore, choice C is correct.

14. The correct answer is A. Every element of set A must have a corresponding element in set B, but element 3 of set A in choice A does not; then it is not a function.

15. The correct answer is A. \(y = x^2 + 1\) moves the graph of \(y = x^2\) up by 1 unit, and therefore choice A is correct.

16. The correct answer is B. If two lines are parallel, the two slopes of these two lines are equal. In choice B, we change the two equations into slope-intercept form first. \(6x + 2y = 5\) is \(y = -3x + \frac{5}{2}\) and \(x + \frac{1}{3}y = 2\) is \(y = -3x + 6\) both have \(-3\) as slope. Therefore, choice B satisfies this condition.

17. The correct answer is C. The first element of the ordered pair must be able to be written in the form of a square of a natural number. Since 4 is the square of the natural number 2, we see that choices A and C may be correct. The second element should belong to set B, since 0.2 = \(\frac{1}{5}\). Then, only choice C is correct.

18. The correct answer is C. The statement \(a \land s\) is a conjunction. In order for it to be true, both \(a\) and \(s\) must be true. If \(x = 2\), then both \(a\) and \(s\) are true.

19. The correct answer is A. Since \(f(-2) = |2(-2) - 5| = |-9| = 9\), then choice A is correct.

20. The correct answer is C. All the area above, \(y = 0.5x\), can be represented by \(y > 0.5x\), and the area below \(y = x + 1\) can be represented by \(y < x + 1\). Then, the shaded area is the area that is above \(y = 0.5x\) and below \(y = x + 1\).

21. The correct answer is C. Since \(\frac{n}{m} > 1\), then \(\left(\frac{n}{m}\right)\left(\frac{n}{m}\right) > \left(\frac{n}{m}\right) > 1\); since \(\frac{m}{n} < 1\), then \(\left(\frac{m}{n}\right)\left(\frac{m}{n}\right) < \left(\frac{m}{n}\right) < 1\).

22. The correct answer is D. By the definition of inequality with absolute value, if \(b > 0\) and \(|y| < b\), then \(-b\)
< y < b. Then in this question: since |x – 2| < 5, then –5 < x – 2 < 5, then –3 < x < 7.

23. The correct answer is A. One way to attack this question is to convert the numbers from base 2 to base 10 and back. “1110” in base 2 is 0(1) + 1(2) + 1(4) + 1(8) = 0 + 2 + 4 + 8 = 14 in base 10. “111” in base 2 is 1(1) + 1(2) + 1(4) = 1 + 2 + 4 = 7 in base 10. Since 14 – 7 = 7, and we know that 7 = 111 in base 2, the correct answer is choice A.

24. The correct answer is B. A is equal to 101 in base two; B is equal to 10,000,000 in base two. Then AB is the result of shifting A to the left by 7 places.

25. The correct answer is C. The last digit of any prime number must be 1, 3, 7, or 9. Then the last digit of A + B must be one of 2, 4, 6, 8, or 0; then A + B at least has 2 as its factor. A – B is not always a composite number, since it may be less than 0, and a composite number must be a natural number, so choice D is wrong.

26. The correct answer is C. $N^3 + 1$ can be factored as $(N + 1)(N^2 – N + 1)$. All the others cannot be factored, so only choice C is always divisible.

27. The correct answer is C. The expression $-7 \leq y \leq 3$ indicates a closed interval. The $\leq$ signs show that –7 and 3 are included in the interval. This can be written as $[-7, 3]$.

28. The correct answer is A. The graph in choice A has the smallest standard deviation. Its bell curve is the steepest. The data points are clustered closely around the mean.

29. The correct answer is B. The mode of the data in the graph is 200. The mode of a set of data is the value that occurs most often in the set. In the graph shown, three of the parades had turnouts of 200 people, and 200 is the value that occurs most frequently.

30. The correct answer is B. Since

$$\frac{1+i}{1-i} = \frac{(1+i)(1+i)}{(1+i)(1-i)} = \frac{(1+i)^2}{(1-i^2)}$$

$$= \frac{(1+i)^2}{2}$$

$$= \frac{1+2i+i^2}{2}$$

$$= \frac{1+2i-1}{2}$$

$$= \frac{2i}{2}$$

$$= i$$

31. The correct answer is B. The interior angles of a triangle add up to 180°. The figure shows that $\angle B$ is a
right angle, so $\angle B$ measures $90^\circ$. The other two angles must add up to $90^\circ$. Therefore, $a = 27^\circ$.

32. The correct answer is A. We factor $y$ completely; then $y = x(x - 1)(x + 1)$. If $x < -1$, then these three terms—$x$, $x - 1$, and $x + 1$—are all less than 0; then their product is less than 0, so choice A is correct.

33. The correct answer is B.

$$16^{x-1} = 2^{4(x-1)}$$

$$2^x = 2^{4(x-1)}$$

$$x^2 = 4(x - 1)$$

$$x^2 - 4x + 4 = 0$$

$$(x - 2)^2 = 0$$

$$x = 2$$

34. The correct answer is D. $x^2 = 25$, so, $x = 5$ or $x = -5$; $y^3 = 64$, so $y = 4$, and $-5 < y$, but $y < 5$, therefore choice D is correct.

35. The correct answer is C. By the definition of logarithm, $\log_2 X = \frac{1}{2}$ can be transformed into the exponential expression $25^{\frac{1}{2}} = X$. Then $X = 5$.

36. The correct answer is B. Since, as indicated by the graph, $a$ must be $0 < a < 1$, and $b > 1$, and $c > 1$, and $c > b$, then $a < b < c$.

37. The correct answer is B. By the Change-of-Base formula, $\log_n m = \frac{\log_e m}{\log_e n}$, we get

$$\log_b X^{\frac{1}{2}} = \frac{\log_a X^{\frac{1}{2}}}{\log_a b}$$

$$= \left(\frac{-1}{2}\log_a X\right)$$

$$= \frac{-1}{5}$$

38. The correct answer is B. Plugging the logic value into $p, a$, we can check which answer is correct. When $p = T, q = T$, then, choices B, C, and D satisfy the Truth Table. When $p = T, q = F$, then only choice B is correct.
39. The correct answer is C. Use the Pythagorean theorem: \(a^2 + b^2 = c^2\). You are given the lengths of the sides of the triangle, 4. Substitute these into the formula to find the length of the hypotenuse:

\[
\begin{align*}
a^2 + b^2 &= c^2 \\
4^2 + 4^2 &= c^2 \\
16 + 16 &= c^2 \\
32 &= c^2 \\
\sqrt{32} &= c \\
\sqrt{16 \times 2} &= c \\
4\sqrt{2} &= c
\end{align*}
\]

40. The correct answer is A. Choice B is the converse statement of "If \(A\) is true, then \(B\) is true." Choice C is the contrapositive statement. Choice D is an undefined statement.

41. The correct answer is D. Let \(W = \) “the weather is good” let \(C = \) “Charles goes to the movies” let \(J = \) “John goes to the movies.” According to the given information, \(W \land C \land J \rightarrow L\). The contrapositive of this statement is \(\sim L \rightarrow \sim (W \land C \land J)\), which can also be written \(\sim L \rightarrow (\sim W \land \sim C \land \sim J)\). Therefore, if Linda did not go to movies, one of the following must be true: the weather is not good, Charles did not go to the movies, or John did not go to the movies. Choice D matches this scenario.

42. The correct answer is D. Since \(A = \) true, and \(B = \) false, then \(A \land B\) is false and \(\sim (A \land B)\) is true, therefore choice A is wrong. Since \(\sim A\) is false, \(\sim B\) is true and \((\sim A) \land \sim B\) is false, then choice B is wrong; Since \(A \lor B\) is true and \(\sim (A \lor B)\) is false, then choice C is wrong; only choice D is correct.

43. The correct answer is C. Plugging the logic values of \(A, B\) into the expressions of the four answers, only choice C satisfies all the relations in this Truth Table.

44. The correct answer is A. If \(A\) is a sufficient condition for \(B\), then \(B\) is a necessary condition for \(A\) According to the conditions, \(A\) and \(C\) have no logical relation, so choices B and C are wrong; \(B\) being a necessary condition for \(C\) cannot guarantee that \(B\) is a sufficient condition for \(C\), so choice D is wrong.

45. The correct answer is B. By the permutation formula, \(n! = n(n-1)...2 \cdot 1\), then 1, 2, 3, and 4 can compose \(4 \cdot 3 \cdot 2 \cdot 1\) permutations, which is equal to 24.

46. The correct answer is B. We have 5 choices for the first letter in a word, 4 choices for the second letter, and 3 for the third. Then we have \(5 \cdot 4 \cdot 3 = 60\) different words with 3 different letters.

47. The correct answer is C. This is a standard problem of combination. By the formula of combination, \(C(n, k) = \frac{n!}{[(n-k)!k!]}\), which means to select \(k\) items from total \(n\) items. We get
48. The correct answer is C. The value of $x$ is $47^\circ$. Perpendicular lines intersect to form $90^\circ$ angles. Therefore, $x + 43^\circ = 90^\circ$.

49. The correct answer is A. According to the Factor Theorem, an expression in the form $x - c$ is a factor of a polynomial function $f(x)$ if the value of $f(c)$ is zero. In other words, if you substitute $c$ for $x$ in the polynomial and the result is zero, then $x - c$ is a factor. Substitute 2 into the polynomial: $7(2)^2 - 12(2) - 4 = 0$. The result is zero, so $x - 2$ is a factor.

50. The correct answer is A. The mean of the sample set is 4, and the median is 2. Then, the difference is 2.

51. The correct answer is A. The total sample space is $(6, 2) = 15$. The number of chances of picking ball $A$ is $(5, 1)(1, 1) = 5$. Then, the probability is $\frac{5}{15} = \frac{1}{3}$.

52. The correct answer is C. Let $P$ represent the event for which we pick the first ball that is not $A$ let $Q$ represent the event for which we pick the second ball that is $A$. Then by the formula for conditional probability $P(Q|P) = \frac{P(QP)}{P(P)}$, events $Q$ and $P$ are independent, and

$$P(QP) = P(Q)P(P)$$
$$= \left(\frac{5}{6}\right)\left(\frac{1}{5}\right)$$
$$= \frac{1}{6}$$

therefore

$$P(QP) = P(Q)P(P)$$
$$= \left(\frac{5}{6}\right)\left(\frac{1}{5}\right)$$
$$= \frac{1}{6}$$

53. The correct answer is C. Since this is a fair die, each of the six numbers $\{1, 2, 3, 4, 5, 6\}$ has equal probability to show. Therefore, 4 points has the probability of $\frac{1}{6}$ to show.
54. The correct answer is D. The total sample space is \{1, 2, 3, 4, 5, 6\}, and each one has the same probability, \(\frac{1}{6}\), to show, and they are independent. Therefore,

\[
P(x \leq 4) = P(1) + P(2) + P(3) + P(4)
\]

\[
= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}
\]

\[
= \frac{2}{3}
\]

55. The correct answer is B. This is an application of a combination problem. Since the polygon has \(N\) sides, then it has \(N\) vertices. Any two vertices have one line, which may be a sideline or diagonal. So we select any 2 from \(N\), which is \((N, 2)\), the number of the diagonals plus the sidelines, then we subtract the \(N\) sidelines. We get the number of diagonals: \((N, 2) - N = 0.5 N(N - 1) - N\).

56. The correct answer is A. In three shots, the probability of hitting exactly twice is \(3, 2)(0.9)^2(0.1) = 3(0.081) = 0.243\); the probability of hitting 3 times is \(3, 3)(0.9)^3 = 0.729\). The total probability is \(0.243 + 0.729 = 0.972\).

57. The correct answer is C. Let \(Q_1\) represent the event that die A shows 1, \(Q_2\) represent the event that die B shows 2. \(Q_1\) and \(Q_2\) are independent. Therefore,

\[
P(Q_1Q_2) = P(Q_1)P(Q_2)
\]

\[
= \frac{1}{6} \times \frac{1}{6}
\]

\[
= \frac{1}{36}
\]

58. The correct answer is A. Let \((n, m)\) represent the points shown by A and B, with \(n\) being the point shown by A, and \(m\) being the point shown by B. Then, \(1, 4\), \((2, 3)\), \((3, 2)\), \((4, 1)\) are all the possible samples that make the sum of the points equal to 5. The probability of each sample is \(\frac{1}{36}\), so the total is \(\frac{4}{36} = \frac{1}{9}\).

59. The correct answer is B. We figure out how many samples satisfy the sum first; for \(P(3)\), we have \(1, 2\), \((2, 1)\); for \(P(7)\), we have \((1, 6)\), \((2, 5)\), \((3, 4)\), \((4, 3)\), \((5, 2)\), \((6, 1)\); for \(P(9)\), we have \((3, 6)\), \((4, 5)\), \((5, 4)\), \((6, 3)\); for \(P(11)\), we have \((5, 6)\), \((6, 5)\). Then we know that \(P(7)\) has the largest number of samples and each sample has the same probability to be shown, so \(P(7)\) is the largest.

60. The correct answer is C. The probability that the 10 components all work is

\[
0.95^{10} = 0.95^5 \times 0.95^5
\]

\[
= 0.77 \times 0.77
\]

\[
= 0.598
\]